

Discussion of
Lars Svensson and Noah Williams
“Bayesian and Adaptive Optimal Policy Under
Model Uncertainty.”

James Bullard

Federal Reserve Bank of St. Louis

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 - ... but has not been a hit with academic economists.
- Why?

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 - They would take optimal actions in the face of the meta-model uncertainty, self-insuring to the extent possible.
 - This would change the equations to which the meta-model uncertainty is being added.
 - This just creates a new model.
- It is not clear that the meta-model concept is the right one for thinking about “model uncertainty.”

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- Markov jump-linear-quadratic system, with forward-looking variables.
- “Modes,” or regimes, follow a Markov process.
- Difference from previous work: modes not observable here.
- Both private sector and policymakers infer modes from their observations of the economy.

Regime switching macroeconomics

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- That system would have a (Bayesian) rational expectations equilibrium.
- That equilibrium may or may not be learnable in standard senses.

Related to MJLQ?

- The authors study equations suggested by an engineering literature:

$$X_{t+1} = A_{11,j+1}X_t + A_{12,j+1}x_t + B_{1,j+1}i_t + C_{1,j+1}\epsilon_{t+1} \quad (1)$$

$$E_t H_{j,t+1} = A_{21,j,t}X_t + A_{22,j,t}x_t + B_{2,j,t}i_t + C_{2,j,t}\epsilon_t \quad (2)$$

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- Do these characterize the REE (Bayesian) equilibrium of an economy with additional, regime-switching uncertainty embedded in the model?
- Experience suggests not, but the authors may be able to provide examples.
- The matrices would not switch like this: instead, agents would take actions based on their inferred probability of being in any particular mode.

Spirit of the analysis

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- Put the emphasis on learning and inference about the probability distribution of modes.
- Assume all agents update their perceived distribution over modes.
- The “REE” occurs when the subjective perceived probability distribution of modes coincides with the exogenous, true distribution.
- Optimal policy takes into account that the distribution of modes is initially unknown.

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- Stability conditions for this process? Expectational stability or related concept?
- How is the REE obtained?
- Some versions may induce a self-confirming equilibrium? Especially when policy choices affect observed modes.

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 - ... but policymakers experiment all the time.
- Adaptive optimal policy as an approximation to Bayesian optimal policy.
- Size of experimentation motive drives differences between BOP and AOP.

Optimal policy with no learning

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- Updating via $p_{t+1|t+1} = P'p_{t|t}$.
- Observations of X_t , x_t , and i_t are not used to update $p_{t|t}$.
- Do the beliefs of the policymaker make sense? Possibly they think modes are independently drawn.

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- Perceived versus true transition equation: Under what conditions do they coincide?
- Expectational stability may be a concern.

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- Cannot ask that question here.

Conclusions

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- Match up with microfoundations is sketchy in this version, and appears difficult.
- BOP only works for small models due to curse of dimensionality.
- AOP a computable alternative, but stability is an open question.
- Remark: There is much greater model uncertainty in the world than is acknowledged in this paper.